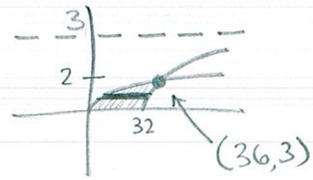
Consider the region bounded by $y=\frac{\sqrt{x}}{3}$, $y=\sqrt{x-32}$ and $y=0$. **SCORE: _____ / 17 PTS**

[a] Suppose the region is revolved around the line $y=3$. Find the volume of the resulting solid.

NOTE: Your final answer must be a number, NOT an integral NOR a sum of integrals.



$y=\frac{\sqrt{x}}{3} \rightarrow x=(3y)^2=9y^2$

$y=\sqrt{x-32} \rightarrow x=y^2+32$

$\boxed{9y^2=y^2+32}$ (1) $\rightarrow 8y^2=32$

$y^2=4$

$y=\pm 2 \rightarrow y=2$

(1) $\boxed{\int_0^2}$ $\boxed{2\pi}$ (1) $(3-y)(y^2+32-9y^2)\,dy$

$= \int_0^2 2\pi \boxed{(3-y)}$ (1) $\boxed{(32-8y^2)}$ (3) dy

$= 16\pi \int_0^2 (3-y)(4-y^2)\,dy$

$= \boxed{16\pi \int_0^2 (12-4y-3y^2+y^3)\,dy}$ (1½)

OK IF YOU DIDN'T FACTOR THE "8" TO THE FRONT OF THE INTEGRAL

$= \boxed{16\pi\left(12y-2y^2-y^3+\frac{1}{4}y^4\right)}\Big|_0^2$ (1½)

$= \boxed{16\pi(24-8-8+4)}$ (1)

$= 16\pi(12)$

$= \boxed{192\pi}$ (1)

[b] Suppose the region is the base of a solid. Cross sections perpendicular to the $x-$axis are isosceles right triangles with their leg in the base region. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

$y=2 \rightarrow x=9(2)^2=36$

(½) $\boxed{\int_0^{32}} \frac{1}{2}\boxed{\left(\frac{\sqrt{x}}{3}\right)^2 dx}$ (1½) $+ \boxed{\int_{32}^{36}}$ (½) $\frac{1}{2}\boxed{\left(\frac{\sqrt{x}}{3}-\sqrt{x-32}\right)^2 dx}$ (1½)

(1) MUST HAVE "$\frac{1}{2}$" IN BOTH INTEGRALS

Find the area between the curves $y = 2e^x$ and $y = 9 - e^x$ over the interval $[0, 4]$.

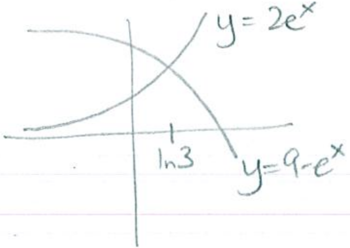
SCORE: _____ / 6 PTS

<u>NOTE: Your final answer must be a number, NOT an integral NOR a sum of integrals.</u>

$2e^x = 9 - e^x$

1 $\boxed{3e^x = 9}$

$x = \ln 3 \approx 1$



$$\int_0^{\ln 3} (9 - e^x - 2e^x)\,dx + \int_{\ln 3}^{4} (2e^x - (9 - e^x))\,dx$$

$$= \boxed{\int_0^{\ln 3}}\boxed{(9 - 3e^x)\,dx} + \boxed{\int_{\ln 3}^{4}}\boxed{(3e^x - 9)\,dx}$$

$\frac{1}{2}$ $\frac{1}{2}$ 1

$$= (\boxed{9x - 3e^x})\Big|_0^{\ln 3} + (3e^x - 9x)\Big|_{\ln 3}^{4}$$

1 1

$$= [9\ln 3 - 9 - (-3)] + [3e^4 - 36 - (9 - 9\ln 3)]$$

$$= \boxed{18\ln 3 + 3e^4 - 51}$$ 1

The region bounded by $y = 4x^2 - 16$, $y = 20x - 41$ and $y = -16$ is revolved around the line $x = -3$. **SCORE: _____ / 7 PTS**

Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the resulting solid **using as few integrals as possible**.

NOTE: Do NOT use geometry shortcuts (ie. volume formulae) to reduce the number of integrals you need to write.

$4x^2 - 16 = 20x - 41$

(1) $\boxed{4x^2 - 20x + 25 = 0}$

$(2x-5)^2 = 0 \rightarrow x = \frac{5}{2}$

$\downarrow$

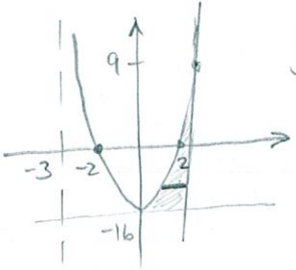
$y = 20 \cdot \frac{5}{2} - 41 = 9$

$y = 4x^2 - 16$

$x = \frac{\sqrt{y+16}}{2}$

$y = 20x - 41$

$x = \frac{y+41}{20}$



(2) FOR BOTH RADII SQUARED AND CORRECT ORDER OF SUBTRACTION

$$\boxed{\int_{-16}^{9}} \boxed{\pi} \boxed{\left(\left(\boxed{\frac{y+41}{20} + 3} \right)^2 - \left(\boxed{\frac{\sqrt{y+16}}{2} + 3} \right)^2 \right) dy}$$

(1) (1) (1) (1)